General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MFP1

Unit Further Pure 1

Friday 14 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The quadratic equation $x^2 - 6x + 18 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find a quadratic equation, with integer coefficients, which has roots α^2 and β^2. *(4 marks)*

(c) Hence find the values of α^2 and β^2. *(1 mark)*

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2 (a) Find, in terms of p and q, the value of the integral $\int_p^q \frac{2}{x^3}\,dx$. *(3 marks)*

(b) Show that only one of the following improper integrals has a finite value, and find that value:

(i) $\int_0^2 \frac{2}{x^3}\,dx$;

(ii) $\int_2^\infty \frac{2}{x^3}\,dx$. *(3 marks)*

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3 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a rotation about the origin through 90° clockwise; *(1 mark)*

(ii) a rotation about the origin through 180°. *(1 mark)*

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

(i) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(ii) Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, for some integer k. *(3 marks)*

(c) Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

(iii) $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

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4 Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

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5 (a) It is given that $z_1 = \frac{1}{2} - \mathrm{i}$.

(i) Calculate the value of ${z_1}^2$, giving your answer in the form $a + b\mathrm{i}$. *(2 marks)*

(ii) Hence verify that z_1 is a root of the equation

$$z^2 + z^* + \frac{1}{4} = 0$$ *(2 marks)*

(b) Show that $z_2 = \frac{1}{2} + \mathrm{i}$ also satisfies the equation in part **(a)(ii)**. *(2 marks)*

(c) Show that the equation in part **(a)(ii)** has two equal **real** roots. *(2 marks)*

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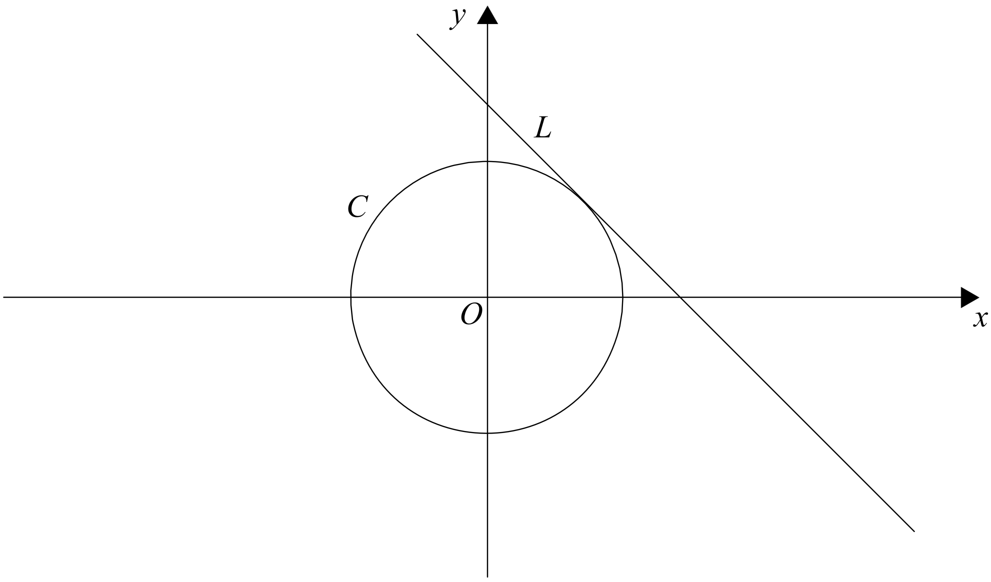
6 The diagram shows a circle C and a line L, which is the tangent to C at the point $(1, 1)$. The equations of C and L are

$$x^2 + y^2 = 2 \quad \text{and} \quad x + y = 2$$

respectively.



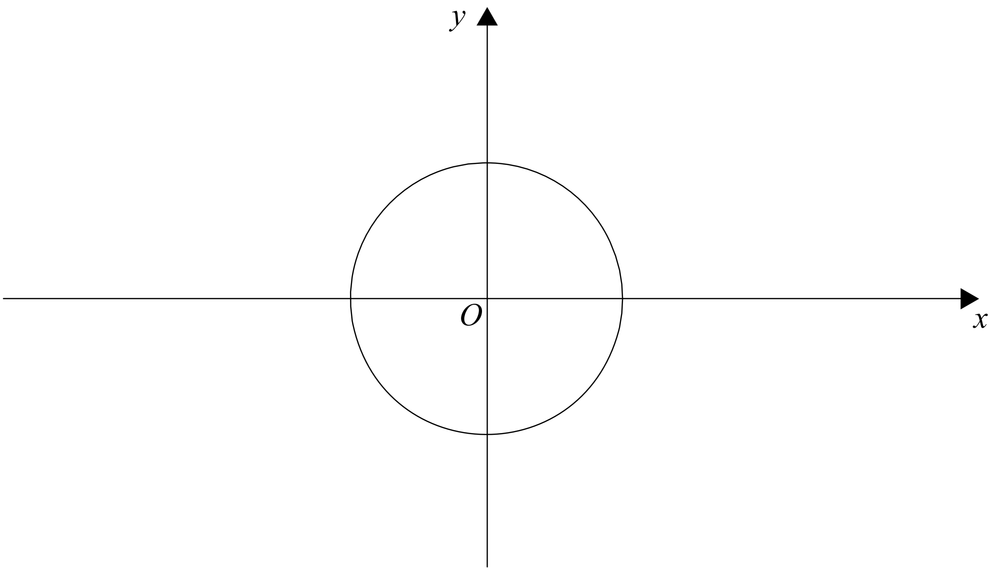
The circle C is now transformed by a stretch with scale factor 2 parallel to the x-axis. The image of C under this stretch is an ellipse E.

(a) **On the diagram below**, sketch the ellipse E, indicating the coordinates of the points where it intersects the coordinate axes. *(4 marks)*

(b) Find equations of:

(i) the ellipse E; *(2 marks)*

(ii) the tangent to E at the point $(2, 1)$. *(2 marks)*



7 A graph has equation

$$y = \frac{x-4}{x^2+9}$$

(a) Explain why the graph has no vertical asymptote and give the equation of the horizontal asymptote. *(2 marks)*

(b) Show that, if the line $y = k$ intersects the graph, the x-coordinates of the points of intersection of the line with the graph must satisfy the equation

$$kx^2 - x + (9k+4) = 0$$ *(2 marks)*

(c) Show that this equation has real roots if $-\frac{1}{2} \leqslant k \leqslant \frac{1}{18}$. *(5 marks)*

(d) Hence find the coordinates of the two stationary points on the curve.

(No credit will be given for methods involving differentiation.) *(6 marks)*

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8 (a) The equation

$$x^3 + 2x^2 + x - 100\,000 = 0$$

has one real root. Taking $x_1 = 50$ as a first approximation to this root, use the Newton-Raphson method to find a second approximation, x_2, to the root. *(3 marks)*

(b) (i) Given that $S_n = \sum_{r=1}^{n} r(3r+1)$, use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$S_n = n(n+1)^2$$ *(5 marks)*

(ii) The lowest integer n for which $S_n > 100\,000$ is denoted by N.

Show that

$$N^3 + 2N^2 + N - 100\,000 > 0$$ *(1 mark)*

(c) Find the value of N, justifying your answer. *(3 marks)*